Neural data analysis - Session 2
Fourier transforms, spectral analysis
Why Fourier?

- A lot of biological data (EEG, circadian rhythms, speech, birdsong, etc.) is oscillatory
- The frequency of these oscillations is often of interest
- Fourier analysis provides a way of identifying the frequencies of the oscillations present in a signal
Math review – complex numbers
Waves - terminology
Fourier series - intuition

- Can represent any periodic function as a sum of sine and cosine functions with different frequencies

\[ f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}. \]

\[ c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)e^{-inx} \, dx. \]
Fourier transforms

- Coefficients in the Fourier series give the “amount” of a particular frequency in the signal.
- From this, one can derive a more general expression describing the frequency content in an arbitrary (non-periodic) signal.
Formal definition

FT of \( f(x) \):

\[
\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) \, e^{-2\pi i x \xi} \, dx,
\]

Inverse transform:

\[
f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) \, e^{2\pi i x \xi} \, d\xi,
\]

Note: Conventions vary widely!!
Discrete Fourier transform

- Fourier transform of a discrete time signal (this is what MATLAB does)
FFT in Matlab

- Fast algorithm for computing the DFT
Power Spectrum
Exercise – sine waves buried in noise
Stationarity, ergodicity

- FT assumes that a signal is stationary – that its frequency content does not change over time
- A related concept is that of ergodicity – that the ensemble average is equivalent to the time average
Windows and the STFT

- Many biological signals are distinctly non-stationary
- Can apply a sliding window to the original signal and compute a new Fourier transform for each window
- Get something called a spectrogram
Exercise – zebra finch song
Exercise 2 – EEG data