

Neural data analysis - Session 2

Fourier transforms, spectral
analysis

Why Fourier?

- A lot of biological data (EEG, circadian rhythms, speech, birdsong, etc.) is oscillatory
- The frequency of these oscillations is often of interest
- Fourier analysis provides a way of identifying the frequencies of the oscillations present in a signal



Math review – complex numbers

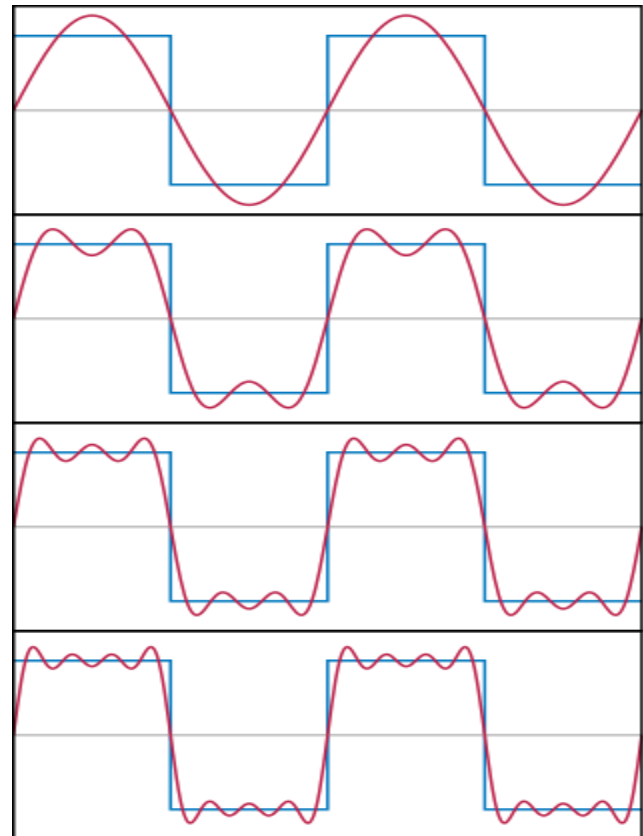
Waves - terminology

Fourier series - intuition

- Can represent any periodic function as a sum of sine and cosine functions with different frequencies

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}.$$

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx.$$



Fourier transforms

- Coefficients in the Fourier series give the “amount” of a particular frequency in the signal
- From this, one can derive a more general expression describing the frequency content in an arbitrary (non-periodic) signal

Formal definition

FT of $f(x)$:

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx,$$

Inverse transform:

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{2\pi i x \xi} d\xi,$$

Note: Conventions vary widely!!

Discrete Fourier transform

- Fourier transform of a discrete time signal
(this is what matlab does)

FFT in Matlab

- Fast algorithm for computing the DFT

Power Spectrum



Exercise – sine waves buried in noise

Stationarity, ergodicity

- FT assumes that a signal is stationary – that its frequency content does not change over time
- A related concept is that of ergodicity – that the ensemble average is equivalent to the time average

Windows and the STFT

- Many biological signals are distinctly non-stationary
- Can apply a sliding window to the original signal and compute a new Fourier transform for each window
- Get something called a spectrogram

Exercise – zebra finch song

Exercise 2 – EEG data